

Axial couplings and strong decay widths of heavy hadrons

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References:

arXiv:1108.5594 [PRD 84, 094502]

arXiv:1109.2480 [PRL 108, 172003]

arXiv:1203.3378 [to appear in PRD]

New Horizons for Lattice Computations with Chiral Fermions
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Heavy-hadron chiral perturbation theory

Effective field theory combining **heavy-quark symmetry** and **chiral symmetry** [M. Wise 1992, T. M. Yan et al. 1992, G. Burdman and J. F. Donoghue 1992, P. Cho 1992]

- pions:

$$\xi = \sqrt{\Sigma} = \exp(i\Phi/f)$$

- heavy-light mesons with $s_l = 1/2$:

$$(\textcolor{red}{P^i}) = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \quad (\textcolor{red}{P^{*i}})_\mu = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}_\mu$$

combined into a single field

$$H^i = [-\textcolor{red}{P^i}\gamma_5 + \textcolor{red}{P_\mu^{*i}}\gamma^\mu] \frac{1-\gamma}{2}$$

Heavy-hadron chiral perturbation theory

- heavy-light baryons with $s_l = 1$:

$$(B^{ij}) = \begin{pmatrix} \Sigma_b^+ & \frac{1}{\sqrt{2}}\Sigma_b^0 \\ \frac{1}{\sqrt{2}}\Sigma_b^0 & \Sigma_b^- \end{pmatrix}, \quad (B^{*ij})_\mu = \begin{pmatrix} \Sigma_b^{*+} & \frac{1}{\sqrt{2}}\Sigma_b^{*0} \\ \frac{1}{\sqrt{2}}\Sigma_b^{*0} & \Sigma_b^{*-} \end{pmatrix}_\mu$$

combined into a single field

$$S_\mu^{ij} = \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma_5 B^{ij} + B_\mu^{*ij}$$

- heavy-light baryons with $s_l = 0$:

$$(T^{ij}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \Lambda_b \\ -\Lambda_b & 0 \end{pmatrix}$$

Heavy-hadron chiral perturbation theory

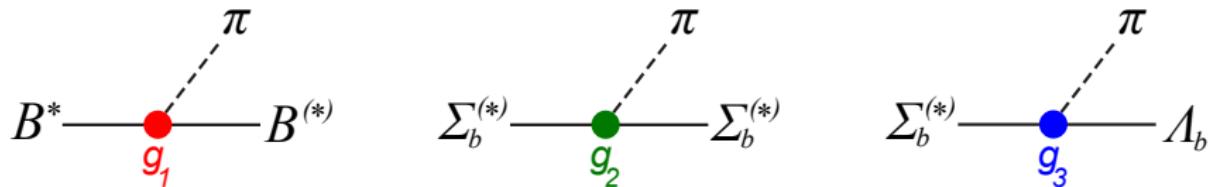
At leading order in chiral- and heavy-quark expansion,

$$\begin{aligned}\mathcal{L} = & \frac{f^2}{8} (\partial^\mu \Sigma^\dagger)_{ij} \partial_\mu \Sigma^{ji} - i \text{Tr} (\bar{H}_i v \cdot \mathcal{D} H^i) - i \bar{S}_{ij}^\mu v \cdot \mathcal{D} S_\mu^{ij} + \Delta \bar{S}_{ij}^\mu S_\mu^{ij} + i \bar{T}_{ij} v \cdot \mathcal{D} T^{ij} \\ & + g_1 \text{Tr} (\bar{H}_i (\mathcal{A}^\mu)_j^i \gamma_\mu \gamma_5 H^j) \\ & - i g_2 \epsilon_{\mu\nu\sigma\lambda} \bar{S}_{ki}^\mu v^\nu (\mathcal{A}^\sigma)_j^i (S^\lambda)^{jk} \\ & + \sqrt{2} g_3 [\bar{S}_{ki}^\mu (\mathcal{A}^\mu)_j^i T^{jk} + \bar{T}_{ki} (\mathcal{A}^\mu)_j^i S_\mu^{jk}],\end{aligned}$$

where

$$\mathcal{A}^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = -\frac{1}{f} \partial^\mu \Phi + \dots$$

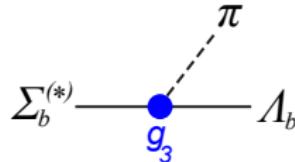
This gives the vertices



$$[g_1 \sim g \sim \hat{g} \sim g_\pi \sim g_{B^* B \pi}]$$

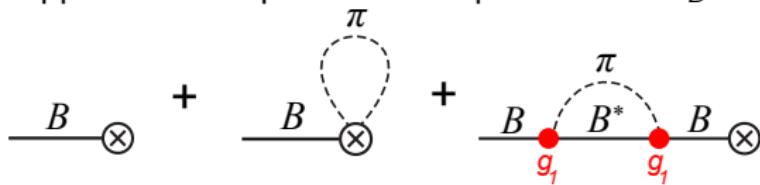
Heavy-hadron chiral perturbation theory

Example application 1: strong decay $\Sigma_b \rightarrow \Lambda_b \pi$



$$\Gamma(\Sigma_b^+ \rightarrow \Lambda_b \pi^+)_{\text{LO}} = \frac{g_3^2}{6\pi f^2} |\mathbf{p}_\pi|^3$$

Example application 2: quark-mass dependence of f_B in $SU(2)$ χ PT:



$$f_B = A \left[1 + \frac{3}{4}(1 + 3g_1^2) \frac{m_\pi^2}{(4\pi f)^2} \log \left(\frac{m_\pi^2}{(4\pi f)^2} \right) \right] + B m_\pi^2$$

Previous knowledge of g_1 , g_2 , g_3 :

Reference	Method	g_1	g_2	g_3
Yan <i>et al.</i> , 1992	Nonrelativistic quark model	1	2	$\sqrt{2}$
Colangelo <i>et al.</i> , 1994	Relativistic quark model	$1/3$
Bećirević, 1999	Quark model with Dirac eq.	0.6 ± 0.1
Guralnik <i>et al.</i> , 1992	Skyrme model	...	1.6	1.3
Colangelo <i>et al.</i> , 1994	Sum rules	0.15 - 0.55
Belyaev <i>et al.</i> , 1994	Sum rules	0.32 ± 0.02
Dosch and Narison, 1995	Sum rules	0.15 ± 0.03
Colangelo and Fazio, 1997	Sum rules	0.09 - 0.44
Pirjol and Yan, 1997	Sum rules	...	$< \sqrt{6 - g_3^2}$	$< \sqrt{2}$
Zhu and Dai, 1998	Sum rules	...	$1.56 \pm 0.30 \pm 0.30$	$0.94 \pm 0.06 \pm 0.20$
Cho and Georgi, 1992	$\mathcal{B}[D^* \rightarrow D \pi, D \gamma]$	0.34 ± 0.48
Arnesen <i>et al.</i> , 2005	$\mathcal{B}[D_{(s)}^* \rightarrow D_{(s)} \pi, D_{(s)} \gamma]$	0.51
Li <i>et al.</i> , 2010	$d\Gamma[B \rightarrow \pi \ell \nu]$	< 0.87
Cheng, 1997	$\Gamma[\Sigma_c^* \rightarrow \Lambda_c \pi]$, NRQM	0.70 ± 0.12	1.40 ± 0.24	0.99 ± 0.17
De Divitiis <i>et al.</i> , 1998	$n_f = 0$ Lattice QCD	0.42 ± 0.09
Abada <i>et al.</i> , 2004	$n_f = 0$ Lattice QCD	0.48 ± 0.11
Negishi <i>et al.</i> , 2007	$n_f = 0$ Lattice QCD	0.517 ± 0.016
Ohki <i>et al.</i> , 2008	$n_f = 2$ Lattice QCD	0.516 ± 0.052
Bećirević <i>et al.</i> , 2009	$n_f = 2$ Lattice QCD	$0.44 \pm 0.03^{+0.07}_{-0.00}$
Bulava <i>et al.</i> , 2010	$n_f = 2$ Lattice QCD	0.51 ± 0.02

This work

Complete calculation of g_1 , g_2 , g_3 from lattice QCD, controlling all systematic uncertainties

- RBC/UKQCD ensembles with 2+1 flavors of domain-wall fermions
- $227 \text{ MeV} \leq m_\pi \leq 352 \text{ MeV}$ (three unitary, three partially quenched)
- $L = 2.7 \text{ fm}$
- NLO $SU(4|2)$ chiral fits including finite-volume effects
- continuum extrapolation using two lattice spacings $a = 0.112 \text{ fm}$,
 $a = 0.085 \text{ fm}$
- chiral symmetry simplifies nonperturbative renormalization, provides automatic $\mathcal{O}(a)$ improvement

How can we calculate g_1 , g_2 , g_3 from QCD?

- We compute suitable hadronic observables both in HH χ PT and in lattice QCD.
- The expressions derived from HH χ PT are then fitted to the lattice data, and in these fits the axial couplings are parameters.
- The simplest quantities that depend on the axial couplings *already at leading order* are the matrix elements of the axial current between single-hadron states.

$$\langle P^{*d} | A_\mu | P^u \rangle = -2 (g_1)_{\text{eff}} \varepsilon_\mu^*,$$

$$\langle S^{dd} | A_\mu | S^{du} \rangle = -(i/\sqrt{2}) (g_2)_{\text{eff}} v^\sigma \epsilon_{\sigma\mu\nu\rho} \bar{U}^\nu U^\rho,$$

$$\langle S^{dd} | A_\mu | T^{du} \rangle = -(g_3)_{\text{eff}} \bar{U}_\mu \mathcal{U}$$

How can we calculate g_1 , g_2 , g_3 from QCD?

In $SU(2)$ chiral perturbation theory and for $m_u = m_d$, one finds

$$(g_1)_{\text{eff}} = g_1 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{4g_1^2}{f^2} H(m_\pi, 0) + \text{analytic terms} \right],$$

$$\begin{aligned} (g_2)_{\text{eff}} = & g_2 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{3g_2^2}{2f^2} H(m_\pi, 0) \right. \\ & \left. + \frac{g_3^2}{f^2} \left\{ H(m_\pi, -\Delta) - 2K(m_\pi, -\Delta, 0) \right\} + \text{analytic terms} \right], \end{aligned}$$

$$\begin{aligned} (g_3)_{\text{eff}} = & g_3 \left[1 - \frac{2}{f^2} I(m_\pi) + \frac{g_2^2}{f^2} \left\{ -2H(m_\pi, \Delta) + H(m_\pi, 0) \right\} \right. \\ & + \frac{g_3^2}{2f^2} \left\{ H(m_\pi, -\Delta) + 9H(m_\pi, \Delta) - 2K(m_\pi, \Delta, 0) \right\} \\ & \left. + \text{analytic terms} \right] \end{aligned}$$

Derived in our paper arXiv:1108.5594, also for $SU(4|2)$, $SU(6|3)$, and in finite volume

Lattice calculation of axial-current matrix elements

Use the interpolating fields

$$\begin{aligned} P^i &= (\gamma_5)_{\alpha\beta} \bar{Q}_{a\alpha} \tilde{q}_{a\beta}^i, \\ P_\mu^{*i} &= (\gamma_\mu)_{\alpha\beta} \bar{Q}_{a\alpha} \tilde{q}_{a\beta}^i, \\ S_{\mu\alpha}^{ij} &= \epsilon_{abc} (C\gamma_\mu)_{\beta\gamma} \tilde{q}_{a\beta}^i \tilde{q}_{b\gamma}^j Q_{c\alpha}, \\ T_\alpha^{ij} &= \epsilon_{abc} (C\gamma_5)_{\beta\gamma} \tilde{q}_{a\beta}^i \tilde{q}_{b\gamma}^j Q_{c\alpha}, \\ A_\mu &= Z_A \bar{d}_{a\alpha} (\gamma_\mu \gamma_5)_{\alpha\beta} u_{a\beta}. \end{aligned}$$

Static heavy quark Q : Eichten-Hill action with HYP smearing

Light quarks u, d, s : domain wall action. Nonperturbative Z_A (thanks to chiral symmetry) from RBC/UKQCD [Aoki et al., 2011]

$$Z_A = \begin{cases} 0.7019(26) & \text{for } a = 0.112 \text{ fm,} \\ 0.7396(17) & \text{for } a = 0.085 \text{ fm} \end{cases}$$

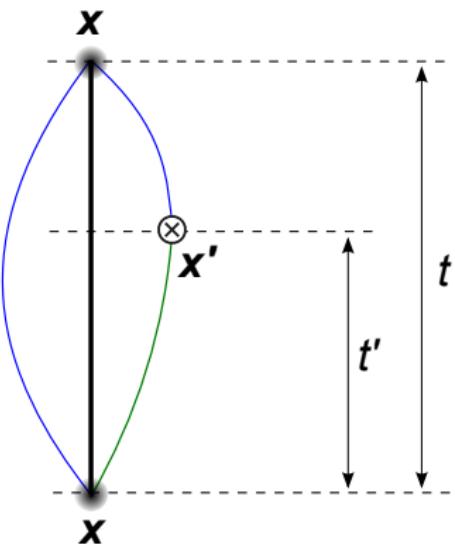
Three-point correlators

For example

$$\langle S^{dd\ \mu}(\mathbf{x}, t) \sum_{\mathbf{x}'} A^\mu(\mathbf{x}', t') \bar{T}_{du}(\mathbf{x}, 0) \rangle$$

Static heavy quark propagator requires hadron interpolating fields to be at same spatial point \mathbf{x} .

We generate pairs of light-quark propagators with sources at $(\mathbf{x}, 0)$ and (\mathbf{x}, t) for multiple values of t .



Correlator ratios

Simple ratios for mass-degenerate $P^* \rightarrow P$ and $S \rightarrow S$ matrix elements

$$R_1(t, t') = -\frac{1}{3} \frac{\sum_{\mu=1}^3 \langle P^{*d\mu}(t) A^\mu(t') P_u^\dagger(0) \rangle}{\langle P^u(t) P_u^\dagger(0) \rangle} \xrightarrow[t, t', |t-t'| \rightarrow \infty]{} (g_1)_{\text{eff}},$$
$$R_2(t, t') = i \frac{\sum_{\mu, \nu, \rho=1}^3 \epsilon_{0\mu\nu\rho} \langle S^{dd\mu}(t) A^\nu(t') \bar{S}_{du}^\rho(0) \rangle}{\sum_{\mu=1}^3 \langle S^{dd\mu}(t) \bar{S}_{dd}^\mu(0) \rangle} \xrightarrow[t, t', |t-t'| \rightarrow \infty]{} (g_2)_{\text{eff}},$$

Double ratio for $S \rightarrow T$ transition matrix element

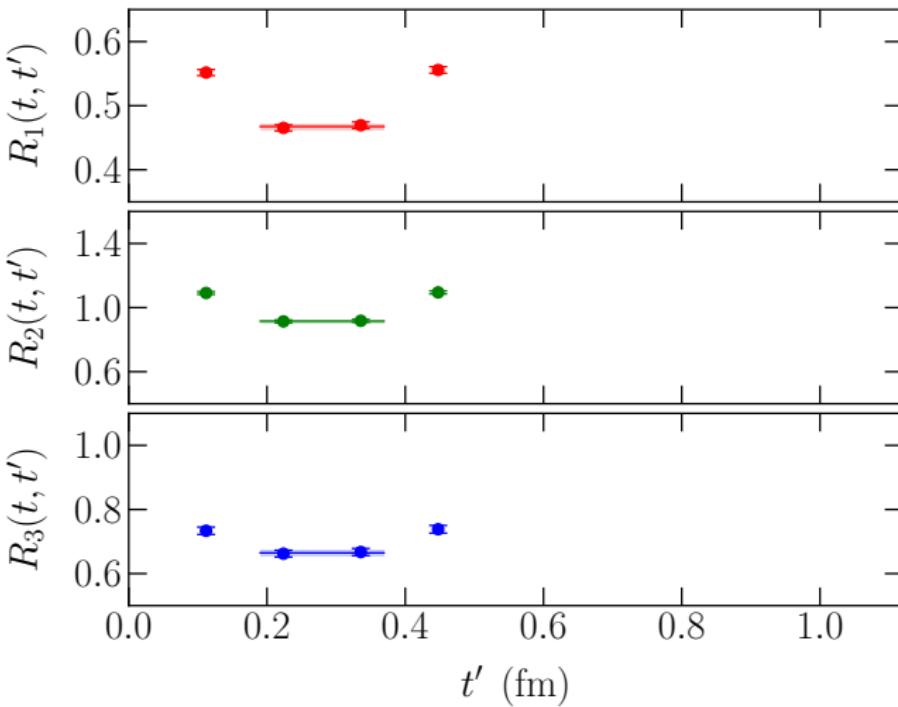
$$R_3(t, t') = \left[\frac{1}{3} \frac{\sum_{\mu, \nu=1}^3 \langle S^{dd\mu}(t) A^\mu(t') \bar{T}_{du}(0) \rangle \langle T^{du}(t) A^{\nu\dagger}(t') \bar{S}_{dd}^\nu(0) \rangle}{\sum_{\mu=1}^3 \langle S^{dd\mu}(t) \bar{S}_{dd}^\mu(0) \rangle \langle T^{du}(t) \bar{T}_{du}(0) \rangle} \right]^{1/2}$$
$$\xrightarrow[t, t', |t-t'| \rightarrow \infty]{} (g_3)_{\text{eff}}$$

Lattice parameters

RBC/UKQCD ensembles with 2+1 flavors of domain wall fermions

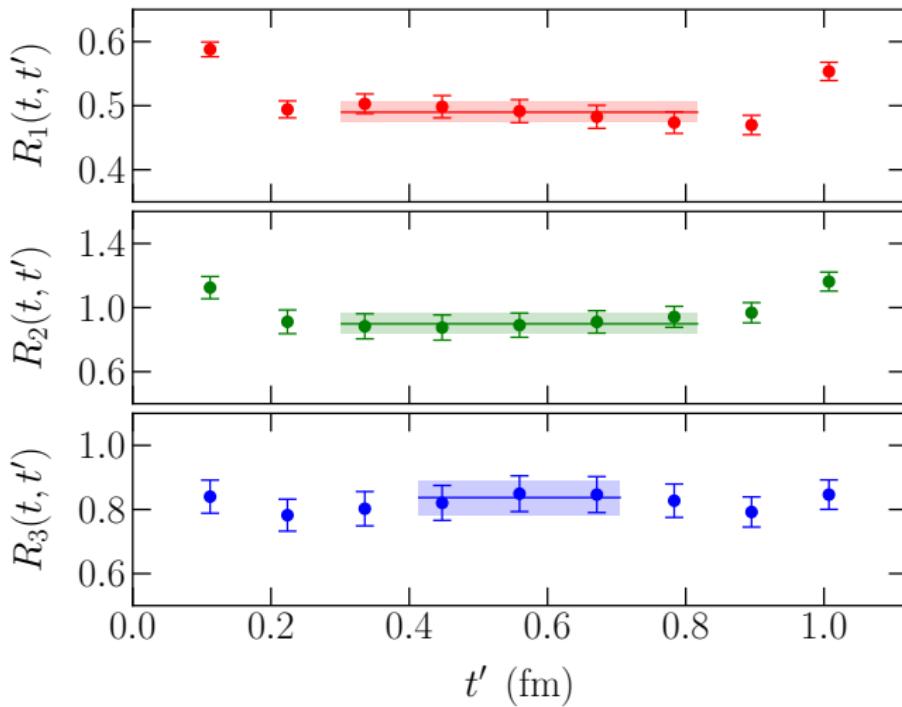
$L^3 \times T$	$m_{u/d}^{(\text{sea})}$	$m_{u/d}^{(\text{val})}$	a (fm)	$m_\pi^{(\text{vs})}$ (MeV)	$m_\pi^{(\text{vv})}$ (MeV)
$24^3 \times 64$	0.005	0.005	0.1119(17)	336(5)	336(5)
$24^3 \times 64$	0.005	0.002	0.1119(17)	304(5)	270(4)
$24^3 \times 64$	0.005	0.001	0.1119(17)	294(5)	245(4)
$32^3 \times 64$	0.006	0.006	0.0848(17)	352(7)	352(7)
$32^3 \times 64$	0.004	0.004	0.0849(12)	295(4)	295(4)
$32^3 \times 64$	0.004	0.002	0.0849(12)	263(4)	227(3)

Examples of ratios



$[a = 0.112 \text{ fm}, am_{u/d}^{(\text{val})} = 0.002, t/a = 5]$

Examples of ratios

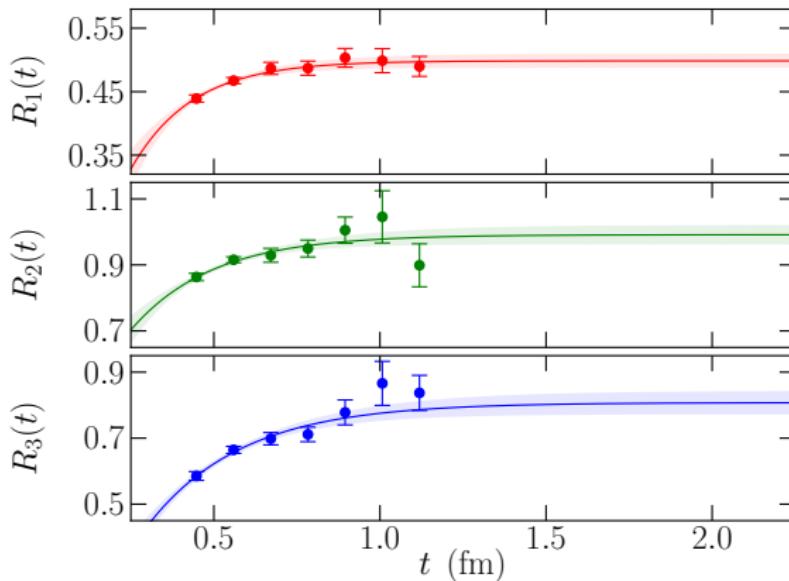


$[a = 0.112 \text{ fm}, am_{u/d}^{(\text{val})} = 0.002, t/a = 10]$

Extrapolation to infinite source-sink separation

Fit t -dependence including first excited-state contamination using

$$R_i(t) = (g_i)_{\text{eff}} - A_i e^{-\delta_i t}$$



$$[a = 0.112 \text{ fm}, am_{u/d}^{(\text{val})} = 0.002]$$

Chiral-continuum fits using $SU(4|2)$ HH χ PT

$$(g_1)_{\text{eff}} = g_1 \left[1 + f_1(g_1, m_\pi^{(\text{vv})}, m_\pi^{(\text{vs})}, L) + d_{1,n_{\text{HYP}}} a^2 + c_1^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_1^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 \right],$$

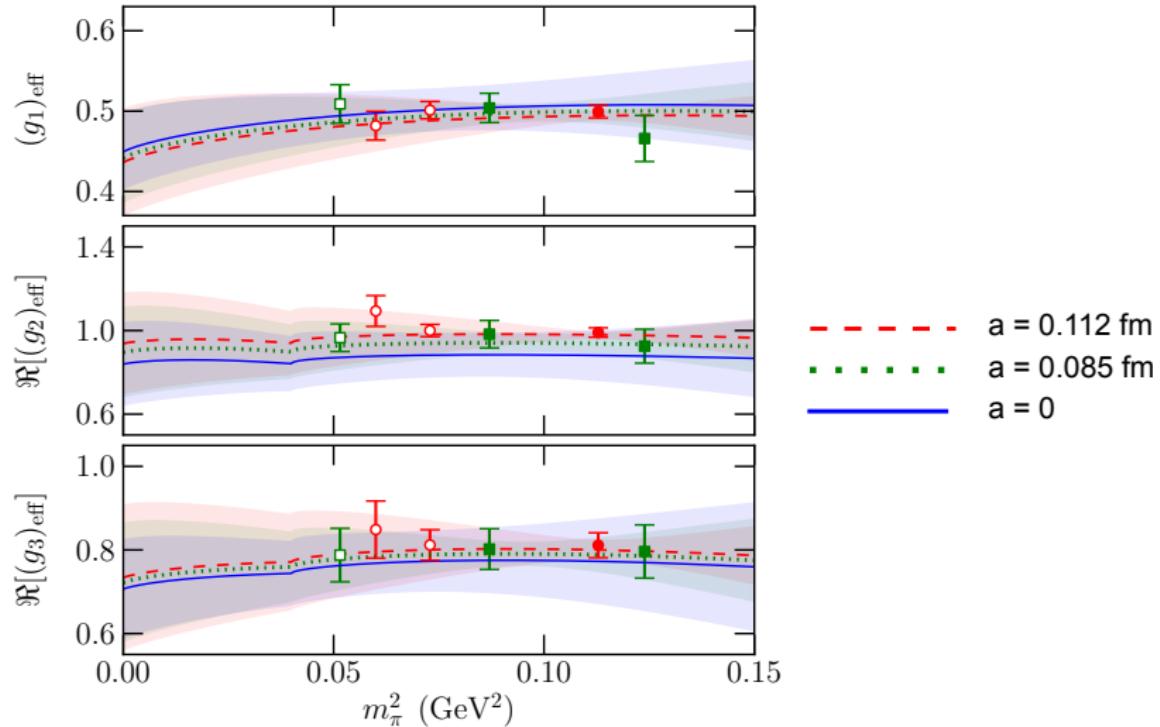
$$(g_2)_{\text{eff}} = g_2 \left[1 + f_2(g_2, g_3, m_\pi^{(\text{vv})}, m_\pi^{(\text{vs})}, \Delta, L) + d_{2,n_{\text{HYP}}} a^2 + c_2^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_2^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 \right],$$

$$(g_3)_{\text{eff}} = g_3 \left[1 + f_3(g_2, g_3, m_\pi^{(\text{vv})}, m_\pi^{(\text{vs})}, \Delta, L) + d_{3,n_{\text{HYP}}} a^2 + c_3^{(\text{vv})} [m_\pi^{(\text{vv})}]^2 + c_3^{(\text{vs})} [m_\pi^{(\text{vs})}]^2 \right]$$

$$(\Delta = 200 \text{ MeV})$$

Fits using $SU(4|2)$ HH χ PT

Plotted at $L = \infty$, $m_\pi^{(\text{vs})} = m_\pi^{(\text{vv})}$:



Final results for axial couplings

Final results:

$$\begin{aligned} g_1 &= 0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}} & = 0.449 \pm 0.051, \\ g_2 &= 0.84 \pm 0.20_{\text{stat}} \pm 0.04_{\text{syst}} & = 0.84 \pm 0.20, \\ g_3 &= 0.71 \pm 0.12_{\text{stat}} \pm 0.04_{\text{syst}} & = 0.71 \pm 0.13. \end{aligned}$$

Systematic uncertainties:

Source	g_1	g_2	g_3
NNLO terms in fits of m_π - and a -dep	3.6%	2.8%	3.7%
Higher excited states in fits to $R_i(t)$	1.7%	2.8%	4.9%
Unphysical value of $m_s^{(\text{sea})}$	1.5%	1.5%	1.5%
Total	4.2%	4.3%	6.3%

g_1 : comparison with previous lattice results

Reference	n_f , action	$[m_\pi^{(vv)}]^2$ (GeV 2)	g_1
De Divitiis <i>et al.</i> , 1998	0, clover	0.58 - 0.81	$0.42 \pm 0.04 \pm 0.08$
Abada <i>et al.</i> , 2004	0, clover	0.30 - 0.71	$0.48 \pm 0.03 \pm 0.11$
Negishi <i>et al.</i> , 2007	0, clover	0.43 - 0.72	0.517 ± 0.016
Ohki <i>et al.</i> , 2008	2, clover	0.24 - 1.2	$0.516 \pm 0.005 \pm 0.033 \pm 0.028$
Bećirević <i>et al.</i> , 2009	2, clover	0.16 - 1.2	$0.44 \pm 0.03_{-0.00}^{+0.07}$
Bulava <i>et al.</i> , 2010	2, clover	0.063 - 0.49	0.51 ± 0.02
This work	2 + 1, DW	0.052 - 0.12	$0.449 \pm 0.047_{\text{stat}} \pm 0.019_{\text{syst}}$

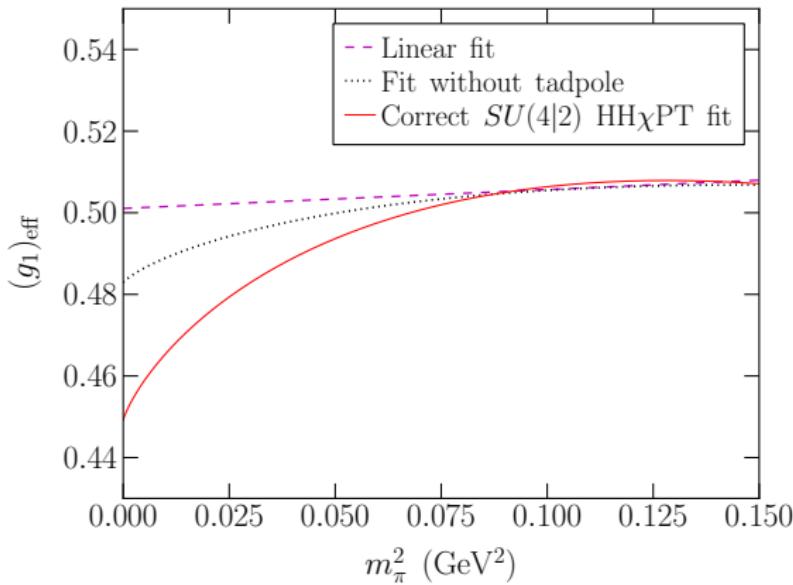
Note: none of the previous works use correct NLO χ PT (**linear** extrapolation or **tadpole missing**)

Impact of NLO χ PT corrections

In $SU(2)$:

Strong decay: $\mathcal{M}(P^* \rightarrow P \pi) \propto g_1 \left[1 - 4g_1^2 \frac{m_\pi^2}{(4\pi f_\pi)^2} \log \frac{m_\pi^2}{\mu^2} + \tilde{c} m_\pi^2 \right].$

Axial current m. elt.: $(g_1)_{\text{eff}} = g_1 \left[1 - (2 + 4g_1^2) \frac{m_\pi^2}{(4\pi f_\pi)^2} \log \frac{m_\pi^2}{\mu^2} + c m_\pi^2 \right].$



Comparison with other results

Reference	Method	g_1	g_2	g_3
Yan <i>et al.</i> , 1992	Nonrelativistic quark model	1	2	$\sqrt{2}$
Colangelo <i>et al.</i> , 1994	Relativistic quark model	1/3
Bećirević, 1999	Quark model with Dirac eq.	0.6 ± 0.1
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Belyaev <i>et al.</i> , 1994	Sum rules	0.32 ± 0.02
Dosch and Narison, 1995	Sum rules	0.15 ± 0.03
Colangelo and Fazio, 1997	Sum rules	0.09 - 0.44
Pirjol and Yan, 1997	Sum rules	...	$< \sqrt{6 - g_3^2}$	$< \sqrt{2}$
Zhu and Dai, 1998	Sum rules	...	$1.56 \pm 0.30 \pm 0.30$	$0.94 \pm 0.06 \pm 0.20$
Cho and Georgi, 1992	$\mathcal{B}[D^* \rightarrow D \pi, D \gamma]$	0.34 ± 0.48
Stewart, 1998	$\mathcal{B}[D_{(s)}^* \rightarrow D_{(s)} \pi, D_{(s)} \gamma]$	$0.27^{+0.04+0.05}_{-0.02-0.02}$
Li <i>et al.</i> , 2010	$d\Gamma[B \rightarrow \pi \ell \nu]$	< 0.87
Cheng, 1997	$\Gamma[\Sigma_c^* \rightarrow \Lambda_c \pi]$, NRQM	0.70 ± 0.12	1.40 ± 0.24	0.99 ± 0.17
This work	Lattice QCD	0.449 ± 0.051	0.84 ± 0.20	0.71 ± 0.13

Our QCD results are MUCH smaller than the quark-model predictions
(even for $g_A^{ud} = 0.75$, as needed to get correct nucleon g_A)

Heavy baryon decays

$S \rightarrow T$ strong decays: leading-order width

At LO in chiral and heavy quark expansion, HH χ PT predicts

$$\Gamma[S \rightarrow T \pi] = c_f^2 \frac{1}{6\pi f_\pi^2} g_3^2 \frac{M_T}{M_S} |\mathbf{p}_\pi|^3$$

where

$$c_f = \begin{cases} 1 & \text{for } \Sigma_Q^{(*)} \rightarrow \Lambda_Q \pi^\pm, \\ 1 & \text{for } \Sigma_Q^{(*)} \rightarrow \Lambda_Q \pi^0, \\ 1/\sqrt{2} & \text{for } \Xi_Q^{(*)} \rightarrow \Xi_Q \pi^\pm, \\ 1/2 & \text{for } \Xi_Q^{(*)} \rightarrow \Xi_Q \pi^0, \end{cases}$$

and

$$|\mathbf{p}_\pi| = \frac{\sqrt{[(M_S - M_T)^2 - m_\pi^2][(M_S + M_T)^2 - m_\pi^2]}}{2M_S}$$

(kinematic factors not expanded in $1/m_Q$ here)

$S \rightarrow T$ strong decays: including $1/m_Q$ correction

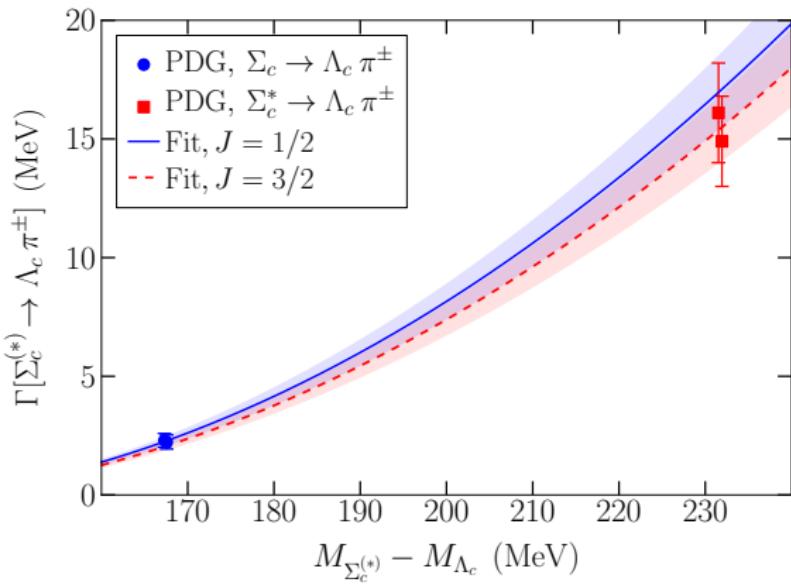
LO in chiral expansion is precise enough, because physical m_π is so small.

LO in HQ expansion is not good enough. Include generic $1/m_Q$ correction

$$\Gamma[S \rightarrow T \pi] = c_f^2 \frac{1}{6\pi f_\pi^2} \left(g_3 + \frac{\kappa_J}{m_Q} \right)^2 \frac{M_T}{M_S} |\mathbf{p}_\pi|^3$$

Determine $\kappa_{1/2}$ and $\kappa_{3/2}$ by fitting experimental data for Σ_c^{++} , Σ_c^0 and Σ_c^{*++} , Σ_c^{*0} with g_3 constrained to LQCD value, and $m_Q = \frac{1}{2} M_{J/\psi}$.

$S \rightarrow T$ strong decays: including $1/m_Q$ correction

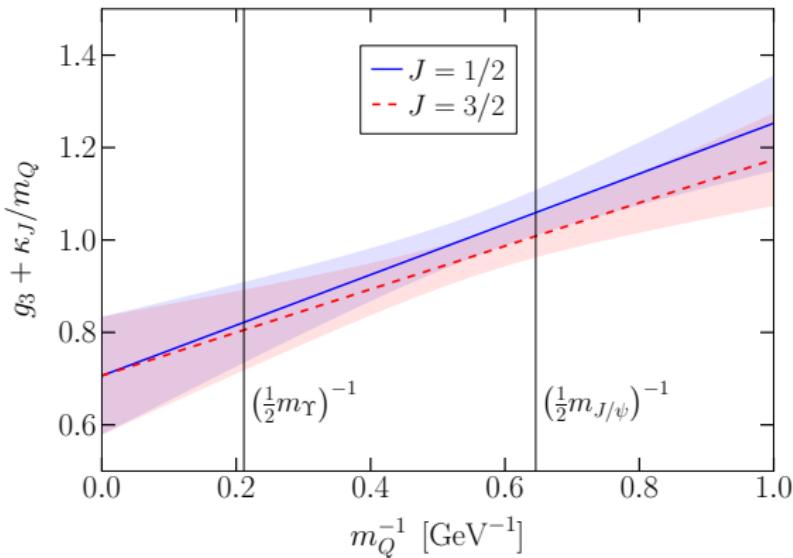


Result:

$$\begin{aligned}\kappa_{1/2} &= 0.55(21) \text{ GeV}, \quad \text{Cov}(\kappa_{1/2}, g_3) = -0.025 \text{ GeV}, \\ \kappa_{3/2} &= 0.47(21) \text{ GeV}, \quad \text{Cov}(\kappa_{3/2}, g_3) = -0.025 \text{ GeV}.\end{aligned}$$

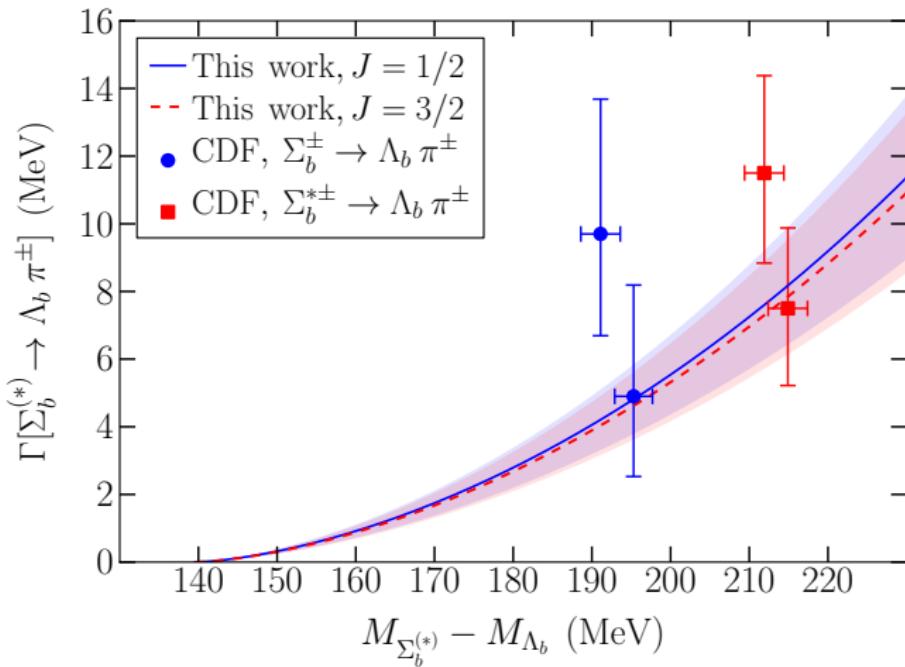
$S \rightarrow T$ strong decays: including $1/m_Q$ correction

Effective coupling vs $1/m_Q$:



→ Evaluate for $m_Q = \frac{1}{2}M_Y$ to make predictions for bottom baryons

$S \rightarrow T$ strong decays: predictions for bottom baryons



Also: Ξ_b^{*0} , recently discovered by CMS

$\Gamma[\Xi_b^{*0}] = 2.1 \pm 1.7$ MeV (CMS), $\Gamma[\Xi_b^{*0}] = 0.51 \pm 0.26$ MeV (this work)

Conclusions

- First complete lattice QCD determination of heavy-hadron axial couplings, Results are $g_1 = 0.449 \pm 0.051$, $g_2 = 0.84 \pm 0.20$, $g_3 = 0.71 \pm 0.13$
- Systematic uncertainties very small, statistical uncertainties by far dominant
- Chiral expansion of axial-current matrix elements well-behaved here, better than for light baryons
- g_1 , g_2 , g_3 are much smaller than NRQM prediction, but ratios close to NRQM
- Compare to $g_A \approx 1.26$, $|g_{N\Delta}| \sim 1.6$ and $g_{\Delta\Delta} \sim -1.9$
- $1/m_Q$ corrections in $\Gamma[\Sigma_Q^{(*)} \rightarrow \Lambda_Q \pi]$ are significant even for $Q = b$
- Calculated $\Gamma[\Sigma_b^{(*)} \rightarrow \Lambda_b \pi]$ widths agree with experiment
- Many future applications (e.g. chiral extrapolations for spectroscopy)

Extra slides

Systematic uncertainties from higher excited states

$$R_i(t) = (g_i)_{\text{eff}} - A_i e^{-\delta_i t}$$

Fit	$(g_1)_{\text{eff}}$	$\delta(g_1)_{\text{eff}}$	$(g_2)_{\text{eff}}$	$\delta(g_2)_{\text{eff}}$	$(g_3)_{\text{eff}}$	$\delta(g_3)_{\text{eff}}$
Original	0.499(11)	0	0.993(29)	0	0.810(36)	0
$t/a = 4$ removed	0.496(13)	0.0030(76)	0.975(35)	0.016(19)	0.783(43)	0.026(15)
$t/a = 4, 5$ removed	0.494(12)	0.0041(76)	0.984(41)	0.009(26)	0.807(54)	0.003(30)
Second exp. added	0.498(11)	0.0009(77)	0.988(30)	0.005(21)	0.796(40)	0.014(36)

Conservative estimate of uncertainty:

$$\sqrt{[\delta(g_i)_{\text{eff}}]^2 + [\sigma \delta(g_i)_{\text{eff}}]^2}$$

$g_1 : 1.7\%$

$g_2 : 2.8\%$

$g_3 : 4.9\%$

Summation Method

Summed ratio:

$$S_i(t) = a \sum_{t'=0}^t R_i(t, t')$$

For large t :

$$S_i(t) \rightarrow c_i + (g_i)_{\text{eff}} t$$

Use slope to extract $(g_i)_{\text{eff}}$:

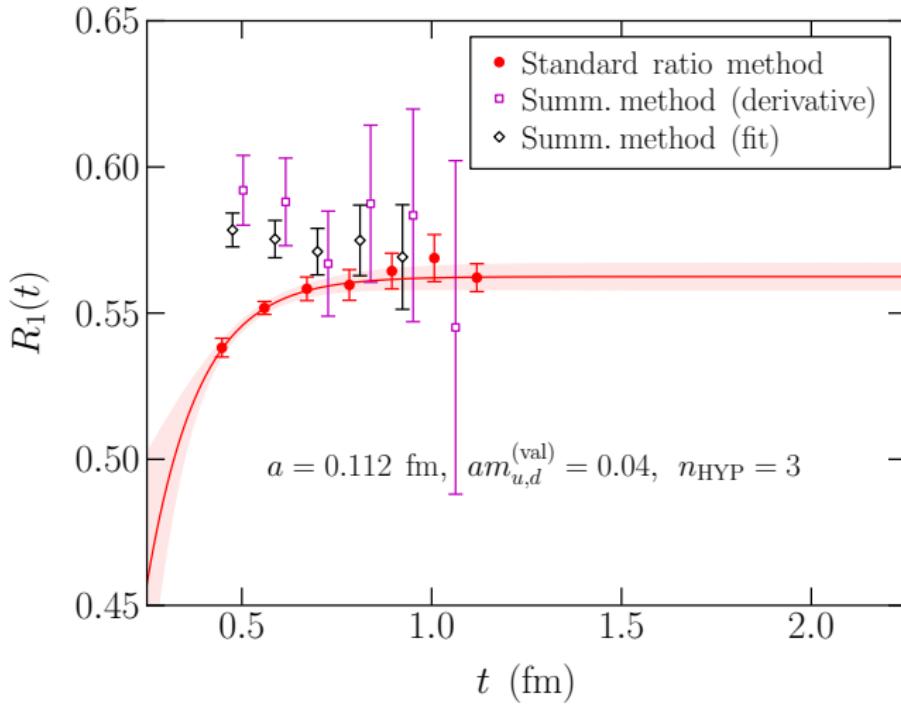
$$R_i^{\text{sum}}(t) = \frac{d}{dt} S_i(t)$$

Expect (if off-diagonal excited state contrib $\neq 0$):

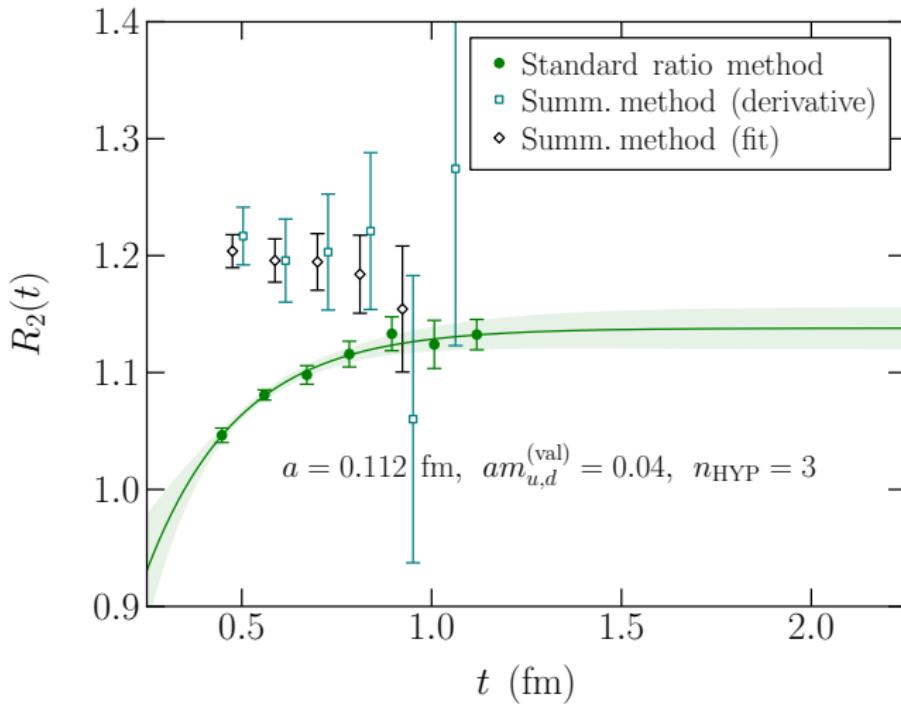
$$\begin{aligned} R_i(t) - (g_i)_{\text{eff}} &= O(e^{-\frac{1}{2}\delta_i t}), \\ R_i^{\text{sum}}(t) - (g_i)_{\text{eff}} &= O(t e^{-\delta_i t}) \end{aligned}$$

[Maiani et al 1987, Gusken et al. 1989, Bulava et al. 2011]

Summation Method



Summation Method



Fits using $SU(4|2)$ HH χ PT: estimates of systematic uncertainties

Add higher-order analytic terms.

$$(g_i)_{\text{eff}}^{(\text{NLO+HO})}(a, m, n_{\text{HYP}})$$

$$\begin{aligned} &= (g_i)_{\text{eff}}^{(\text{NLO})}(a, m, n_{\text{HYP}}) \\ &\quad + g_i \left[c_i^{(\text{vv}, \text{vv})} [m_\pi^{(\text{vv})}]^4 + c_i^{(\text{vs}, \text{vs})} [m_\pi^{(\text{vs})}]^4 + c_i^{(\text{vv}, \text{vs})} [m_\pi^{(\text{vv})}]^2 [m_\pi^{(\text{vs})}]^2 \right. \\ &\quad \left. + d_{i, n_{\text{HYP}}}^{(\text{vv})} a^2 [m_\pi^{(\text{vv})}]^2 + d_{i, n_{\text{HYP}}}^{(\text{vs})} a^2 [m_\pi^{(\text{vs})}]^2 + h_{i, n_{\text{HYP}}} a^4 \right] \end{aligned}$$

Fits using $SU(4|2)$ HH χ PT: estimates of systematic uncertainties

Constrain with Gaussian priors. In terms of the natural scales,

$$c_i^{(vv,vv)} = 0 \pm w/\Lambda_\chi^4,$$

$$c_i^{(vs,vs)} = 0 \pm w/\Lambda_\chi^4,$$

$$c_i^{(vv,vs)} = 0 \pm w/\Lambda_\chi^4,$$

$$d_{i, n_{\text{HYP}}}^{(vv)} = 0 \pm w \Lambda_{\text{QCD}}^2/\Lambda_\chi^2,$$

$$d_{i, n_{\text{HYP}}}^{(vs)} = 0 \pm w \Lambda_{\text{QCD}}^2/\Lambda_\chi^2,$$

$$h_{i, n_{\text{HYP}}} = 0 \pm w \Lambda_{\text{QCD}}^4.$$

w is the width

Fits using $SU(4|2)$ HH χ PT: estimates of systematic uncertainties

w	g_1	$\delta\sigma(g_1)$	g_2	$\delta\sigma(g_2)$	g_3	$\delta\sigma(g_3)$
0	0.449(47)	0	0.84(20)	0	0.71(12)	0
1	0.449(47)	0.0020	0.84(20)	0.0023	0.71(12)	0.0045
5	0.452(48)	0.0089	0.84(20)	0.014	0.70(12)	0.017
10	0.455(50)	0.016	0.84(20)	0.024	0.70(12)	0.026
50	0.464(72)	0.054	0.82(22)	0.099	0.68(15)	0.094
100	0.452(94)	0.082	0.78(26)	0.17	0.63(21)	0.17

Conservative estimate: with $w = 10$, use

$$\delta\sigma(g_i) = \sqrt{\sigma^2(g_i)^{(\text{NLO+HO})} - \sigma^2(g_i)^{(\text{NLO})}}$$

g_1 : 3.6%

g_2 : 2.8%

g_3 : 3.7%

Fits using $SU(4|2)$ HH χ PT: finite-volume corrections

$m_\pi^{(\text{vs})}$ (MeV)	$m_\pi^{(\text{vv})}$ (MeV)	$\frac{(g_1)_{\text{eff}}^{(\infty)} - (g_1)_{\text{eff}}^{(L)}}{(g_1)_{\text{eff}}^{(\infty)}}$	$\frac{(g_2)_{\text{eff}}^{(\infty)} - (g_2)_{\text{eff}}^{(L)}}{(g_2)_{\text{eff}}^{(\infty)}}$	$\frac{(g_3)_{\text{eff}}^{(\infty)} - (g_3)_{\text{eff}}^{(L)}}{(g_3)_{\text{eff}}^{(\infty)}}$
294	245	0.0057	0.015	0.0074
304	270	0.0040	0.0070	0.0027
336	336	0.0016	0.00037	-0.00079
263	227	0.0072	0.028	0.013
295	295	0.0031	0.00027	-0.0012
352	352	0.0013	0.00033	-0.00071